HADRON DIFFRACTION DISSOCIATION AND THE TRIPLE POMERON VERTEX

R. Fiore[a†], L. L. Jenkovszky[b‡], F. Paccanoni[c*]

[a] *Dipartimento di Fisica, Università della Calabria ,*
Istituto Nazionale di Fisica Nucleare, Gruppo collegato di Cosenza
Arcavacata di Rende, I-87030 Cosenza, Italy

[b] *Bogoliubov Institute for Theoretical Physics,*
Academy of Sciences of the Ukrain
252143 Kiev, Ukrain

[c] *Dipartimento di Fisica, Università di Padova,*
Istituto Nazionale di Fisica Nucleare, Sezione di Padova
via F. Marzolo 8, I-35131 Padova, Italy

Abstract

Hadron diffraction dissociation is considered in the dipole Pomeron model where the Pomeron is represented by a double pole in the J-plane. We find that unitarity is satisfied without decoupling of the triple Pomeron vertex. The reaction $\bar{p} + p \to \bar{p} + X$ is analysed.

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[†] *email address:* FIORE @CS.INFN.IT
[‡] *email address:* JENK @GLUK.APC.ORG
[*] *email address:* PACCANONI @PADOVA.INFN.IT

1. INTRODUCTION

The inclusive process of hadron diffraction dissociation has been discussed extensively in the literature [1, 2, 3, 4, 5, 6]. In this paper we will be mainly concerned with the violation of unitarity appearing in the Regge theory, where the diffraction cross-section rises faster than the total one.

Solutions to this problem have been proposed based on different unitarization recipes. Eikonal corrections [7] succeded, for example, in reproducing the main features of single diffraction at high energy obtaining a weak energy dependence of the relevant cross-sections. The inclusion of cuts in the Regge theory [8] has the same effect while a different approach, based on the unitarization of the Pomeron flux, has been advocated by [9] and [10]. Alternative ways of unitarization have been considered in [11]. All the above approaches are based on a supercritical Pomeron input.

A different solution is provided by successful models that are based on the following assumption [12]. Suppose that asymptotically the absorptive part in the s-channel, A(s,t), goes like

$$A(s,t) \propto \beta_1(t)\beta_2(t)s^{\alpha(t)}[h(t)\ln s + C] \ ,$$

then the partial wave amplitude presents a simple and a double pole in the complex J-plane. The amplitude for the Pomeron exchange can then be written as

$$T(s,t) \propto -\frac{(-is)^{\alpha(t)}}{\sin(\pi\alpha(t)/2)}\beta_1(t)\beta_2(t)[h(t)(\ln s - i\frac{\pi}{2}) + C] \ , \qquad (1)$$

where constant terms have been collected in C.

Eq. (1) derives from an ansatz on the form of the Regge residues and different expressions, as far as the t-dependence of h(t) is concerned, can be found in the literature. As an example, in a dual model, if the residue of the simple pole has the

form $\beta(\alpha(t))$, the residue of the double pole will be given by $\int \beta(\alpha(t))\, d\alpha + const.$ This formalism gives an excellent description of $p-p$ and $p-\bar{p}$ elastic scattering including the dip [12]. We will show that, starting from Eq .(1), it is possible to satisfy unitarity at the Born level, without eikonalization.

In the following Sections we apply this model to single diffractive dissociation following, and developing further, the previous work [13] on this subject.

2. DIFFRACTIVE DISSOCIATION

Consider first the process $a+b \to c+X$ with the exchange of Regge trajectories i. From the Mueller discontinuity formula [14] we get

$$\pi E_c \frac{d^3\sigma}{d\vec{p}_c} = \frac{1}{16\pi s} \sum_X \left| \sum_i \beta^i_{a\bar{c}}(t)\xi_i(t) F^{ib\to X}(M^2,t) \left(\frac{s}{M^2}\right)^{\alpha_i(t)} \right|^2 \tag{2}$$

in the usual Regge pole model. M^2 is the squared mass of the unrevealed state X and $\alpha_i(t)$ represents the Regge trajectory exchanged. In the following $i = P, f, \pi$, while the ω trajectory will be neglected on the basis of historical fits [1, 15]. P stands for the Pomeron trajectory and $\xi_i(t) = (1 \pm \exp(-i\pi\alpha_i(t))/\sin(\pi\alpha_i(t))$ is the signature.

In the dipole Pomeron approach, Eq. (2) becomes

$$\pi E_c \frac{d^3\sigma}{d\vec{p}_c} = \\ \frac{1}{16\pi s} \sum_X \left| \beta^P_{a\bar{c}}(t) \left(-i\frac{s}{M^2}\right)^{\alpha_P(t)} \left[h(t)(\ln\frac{s}{M^2} - i\pi/2) + C \right] F^{Pb\to X}(M^2,t) + \right. \\ \left. \sum_{i\neq P} \beta^i_{a\bar{c}}\xi_i(t) F^{ib\to X}(M^2,t) \left(\frac{s}{M^2}\right)^{\alpha_i(t)} \right|^2 . \tag{3}$$

Since π contributes in a different kinematical region with respect to P and f, interference terms between π and P, f are suppressed. Hence in Eq. (3) the sum

over i refers only to f, and the π contribution will be chosen as in [6, 15, 16, 18]:

$$\frac{1}{4\pi}\frac{g^2_{\pi ac}}{4\pi}\frac{(-t)}{(t-\mu^2)^2}\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)-1} G^2(t)\sigma_T^{\pi p}(M^2) \ , \tag{4}$$

where

$$G(t) = \frac{2.3-\mu^2}{2.3-t} \ .$$

Since $a = c = \bar{p}$ and $b = p$, we have also

$$\frac{g^2_{\pi pp}}{4\pi} = 14.6 \ .$$

The f-exchange can be treated in the approximation suggested in [19, 20] with the result that the s-dependence will be modified in Eq. (3) as

$$\left(\frac{s}{M^2}\right)^{2\alpha_P(t)} [(h(t)\ln\frac{s}{M^2}+C)^2 + \frac{\pi^2}{4}h^2(t) + R(s,t)] \ , \tag{5}$$

where

$$\begin{aligned} R(s,t) \ &= \ k\left([h(t)\ln\frac{s}{M^2}+C]\cos\left(\frac{\pi a(t)}{2}\right) - \right. \\ &\left. \frac{\pi h(t)}{2}\sin\left(\frac{\pi a(t)}{2}\right)\right)\left(\frac{s}{M^2}\right)^{-a(t)} + k^2\left(\frac{s}{M^2}\right)^{-2a(t)} \end{aligned} \tag{6}$$

and $a(t)$ is the difference between the P and f trajectories:

$$a(t) = \alpha_P(t) - \alpha_f(t) = a(0) - \delta t.$$

Typical values are $a(0) \simeq 0.5$ and $\delta \simeq 0.65$. In [19] it is quoted for k a value near 7.8.

3. THE TRIPLE POMERON

Let us consider now the triple Pomeron contribution to Eq. (3), with $\alpha_P(t) = 1 + \alpha' t$ and $\alpha' = 0.25\, GeV^{-2}$,

$$\begin{aligned} &\frac{1}{16\pi s}[\beta^P_{a\bar{c}}(t)]^2\left(\frac{s}{M^2}\right)^{2\alpha_P(t)} \times \\ &\left[(h(t)\ln\frac{s}{M^2}+C)^2 + \frac{\pi^2}{4}h^2(t)\right] Im\, T^{Pb}(M^2,t,\alpha_P(t),t_{b\bar{b}}=0) \ , \end{aligned} \tag{7}$$

where, according to the dipole Pomeron model,

$$Im\, T^{Pb} = \sigma_0\, (M^2)^{\alpha_P(0)}(\lambda + h(0) \ln M^2) g(t) \ , \tag{8}$$

$g(t)$ being the triple Pomeron coupling and, for simplicity sake, the same function $h(t)$ has been considered. A term, decreasing with M^2, could well be present in (8) if we consider also secondary trajectories in $P-b$ scattering. Hence Eq. (8) will be valid only for M^2 sufficiently large.

The presence of the function $h(t)$ in the contribution (7) and Eq. (8) is characteristic of the model considered. In terms of partial waves, if

$$\frac{d}{dJ} \left[\beta^P_{a\bar{c}}(J,t) F^{Pb\to X}(J, M^2, t)\right]_{J=\alpha_P(t)} \tag{9}$$

is the coefficient of the simple pole, then the coefficient of $\ln s$,

$$\beta^P_{a\bar{c}}(J,t) F^{Pb\to X}(J, M^2, t)\Big|_{J=\alpha_P(t)} \ , \tag{10}$$

can be obtained from the expression (9) by integration provided a phenomenological form for the residue of the simple pole is available. In absence of a reliable input we must resort to other constraints, for example we can impose that unitarity is satisfied.

By integrating Eq. (3) over t and M^2 we get the single diffractive cross-section, σ_{SD}. The constraint $\sigma_{SD} < \sigma_T$ for all values of s requires that $h(t) \propto (-t)^\gamma$. A lower bound for γ will be discussed in the next Section. Anyway, explicit examples where $h(t)$ must vanish as $(-t)$ when t goes to zero, can be readily found. Assuming the simple form $h(t) = h(-t)^\gamma$,with h constant, and taking the phenomenological expression $\exp(bt)$ for the Pomeron-proton vertex, the triple Pomeron contribution (7) becomes

$$\frac{h^2 \sigma_0 M^2 \lambda g(0)}{16\pi s} e^{2(bt+\alpha_P(t)\ln(s/M^2))} \left[((-t)^\gamma \ln\frac{s}{M^2} + C)^2 + \frac{\pi^2}{4}(-t)^{2\gamma}\right] \ , \tag{11}$$

where the triple Pomeron vertex has been considered as constant, according to experiments [6, 17]. C/h has been renamed as C.

The final form of the differential cross-section is

$$\begin{aligned}\frac{d^2\sigma}{dt\,dM^2} &= \frac{A}{M^2}e^{2(b+\alpha'\ln(s/M^2))t}\left[((-t)^\gamma\ln\frac{s}{M^2}+C)^2+\frac{\pi^2}{4}(-t)^{2\gamma}+R(s,t)/h^2\right]+\\ &\quad \frac{1}{4\pi}\frac{g^2}{4\pi M^2}\frac{(-t)}{(t-\mu^2)^2}G^2(t)\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)-2}\sigma_T^{\pi p}(M^2)\ , \qquad (12)\end{aligned}$$

with

$$A=\frac{h^2\sigma_0\lambda g(0)}{16\pi}\ .$$

In $R(s,t)$ the substitution $h(t)=h\,(-t)^\gamma$ must be performed and C has been redefined accordingly. As far as the other parameters are concerned, b will be fixed from p-p elastic scattering, e.g. $b=2.25\,GeV^{-2}$, and $\sigma_T^{\pi p}(M^2)$ in the dipole Pomeron model can be written as

$$\sigma_T^{\pi p}(M^2)=3.62+2.55\ln(M^2)+38.89(M^2)^{-0.34} \qquad (13)$$

inspired by the parametrization used in [21]. Moreover

$$\alpha_\pi(t)=0.9\,t\ .$$

Since the form of $h(t)$ is determined only near $t=0$, it is well possible that the t-dependence of the cross-section should be corrected. Hence a different value of b could be required from the experimental data.

4. THE PARAMETER γ AND ITS LOWER BOUND

In order to make explicit the unitarity constraint on the single diffraction cross-section we must integrate Eq. (12) in the variables t and M^2. By integrating in the variable t in the range $(-\infty,0)$ we get the contribution to $d\sigma/dM^2$ from the pion and f trajectories given in Appendix.

More important for the bound is the integral over the same t-interval of the Pomeron contribution 11:

$$\frac{A}{M^2}\left[\Gamma(2\gamma+1)\frac{(\ln s/M^2)^2+\pi^2/4}{p^{2\gamma+1}}+2\Gamma(\gamma+1)C\frac{\ln s/M^2}{p^{\gamma+1}}+\frac{C^2}{p}\right] , \tag{14}$$

where

$$p = 2\left(b+\alpha'\ln\frac{s}{M^2}\right) .$$

When integrated over M^2, between the limits $\zeta = M^2_{min}$ and ρs [1], the expression (14) gives

$$A\int_{\ln(1/\rho)}^{\ln(s/\zeta)} dx\left[\Gamma(2\gamma+1)\frac{x^2+\pi^2/4}{(2(b+\alpha'x))^{2\gamma+1}} \quad + \right.$$
$$\left. 2\Gamma(\gamma+1)C\frac{x}{(2(b+\alpha'x))^{\gamma+1}} \quad + \quad \frac{C^2}{2(b+\alpha'x)}\right] , \tag{15}$$

whose increase must be bounded by $\ln s$, that is the behaviour in s of the total cross-section in the dipole Pomeron model. The smallest allowed value of γ is $\gamma_{min.} \geq 1/2$. This can be easily seen by noticing that

$$\int_l^u dx\frac{x^n}{(1+ax)^\nu} = \frac{1}{a^{n+1}}\sum_{m=0}^{n}\left(\begin{array}{c} n \\ m \end{array}\right)\frac{1}{m-\nu+1}(1+ax)^{m-\nu+1}\Big|_l^u$$

has the asymptotic behaviour $u^{n-\nu+1}+O(u^{n-\nu})$, for ν not integer, and that $\nu-n \geq 0$.

Hence, the parameter γ, in general, must satisfy: $\gamma \geq 1/2$. This inequality is necessary to avoid terms, violating unitarity, that rise faster than $\ln s$. It is important to notice that the triple Pomeron contribution does not vanish at $t=0$ because of the presence of the constant C.

5. COMPARISON WITH DATA AND CONCLUSIONS

When comparing the model with experimental data we find two kinds of problems. The first one is related to the experimental definition of single diffraction

[1] CDF [22] chooses $\zeta = 1.4\,GeV^2$ and $\rho = 0.15$.

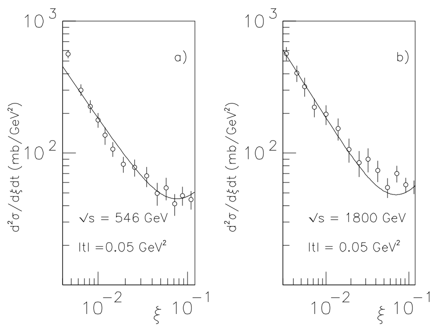

Figure 1: Differential cross sections $d^2\sigma/d\xi\, dt$ vs ξ, data are from [22] compiled by [17]. The solid curves represent the result of the model.

dissociation. The great variety of phenomenological models adopted from different experimental groups, in order to extract the published data, makes difficult the test of any new model. Moreover, integrated cross-section do not refer to the same intervals of M^2 and t, for different experimental analyses.

The second kind of problem resides in our parametrization and is strongly related to the first one. While the pion contribution can be fixed as in Section 2, the parameters relative to the f trajectory must be refitted since the Pomeron contribution is quite different from the one proposed in [19, 20]. Moreover, the parameter γ requires fine tuning, since its value must be larger than $\frac{1}{2}$, and the choice of the function $h(t)$ has been made only on the basis of its small t behaviour.

In view of these difficulties, we simplify the analysis by neglecting the f-contri bution together with the $P - f$ interference term. The π contribution has no free parameters and we are left with three parameters for the Pomeron and a possible correction to the slope b.

By comparing the slope of $d^2\sigma/d\xi dt$, at fixed $\xi = M^2/s = 0.035$ and in the

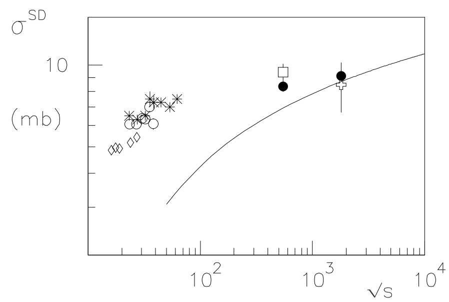

Figure 2: Total single diffraction cross section vs $\sqrt{s}$ compared with the prediction of the model. Data are taken from the compilation of [17].

interval $0.05 < -t < 0.15$, with the CDF [22] and UA8 [18] best fits, we find that at $\sqrt{s} = 1800\, GeV$ the prediction of the model is well within the experimental errors if we choose $\gamma = 0.95$ and keep the previous value for b, $b = 2.25\, GeV^{-2}$. At $\sqrt{s} = 546\, GeV$ the slope agrees with [18] and simulates the effect of a non linear Pomeron trajectory [18, 24]. Larger values of the parameter γ are compatible with the experimental slope and bring forth a flatter total diffractive cross section, as can be seen from the Pomeron contribution (15).

Then we performed a fit, at fixed $|t| = 0.05\, GeV^2$, of the CDF data [22], taken from the providential compilation in [17]. Figs. 1a and 1b show that the proposed model is adequate as far as the ξ dependence is concerned. Only the data above the resonance region, $M^2 > 5\, GeV^2$, and in the coherence region [6], $M^2/s < 0.1$, have been considered. The parameters of the fit are $A = 0.1098 and\, C = 4.557$, with strong correlations between them.

If we consider the data at smaller s, $\sqrt{s} = 20\, GeV$, [23] compiled in [17], the calculated cross section is well below the experimental points. We attribute such a

discrepancy to the neglect of the f trajectory that, in this energy region, plays an important role in $p-\bar{p}$ interaction. The f contribution could also modify sensibly the imaginary part of the $P-b$ amplitude in Eq. (3).

In Fig. 2 the total single diffractive cross section σ_{sd}, for the process $p(\bar{p})+p \to p(\bar{p})+X$, is shown as a function of $\sqrt{s}$. We have considered the experimental data of [22, 25, 26, 27, 28] from the compilation given in [17] where some data have been corrected in order to obtain the diffraction cross section for $\xi \leq 0.05$. Again, the absence of the f trajectory makes the cross section too small at low energy. Analogous results have been obtained in a theoretical model [7] and can be deduced from a phenomenological parametrization [22]. We notice that the cross-section rises as $(\ln s)^{0.1}$ in this model. A more complete fit of all the data will be considered elsewhere.

From the theoretical pointof view, the result in Eq. (12) possesses two important properties that seem required by the data [17]. First, exact factorization, typical of the Regge pole model, is lost in the dipole Pomeron approach. Second, for $t=0$, the Pomeron and pion contributions are independent of s. Finally we remark that this model respects the unitarity condition without decoupling of the triple Pomeron vertex. The total diffractive cross section presents a slower rise than the total $p-\bar{p}$ cross section that, in turn, satisfies the Froissart bound.

1 Appendix

By evaluating the integral over t of the expression (4) in the range $(-\infty, 0)$ we get the contribution of the π-trajectory to $d\sigma/dM^2$:

$$\frac{1}{4\pi}\left(\frac{g^2}{4\pi}\right)\frac{1}{M^2}\left(\frac{s}{M^2}\right)^{-2}\sigma_T^{\pi p}(M^2)\left[e^{\mu^2 p_\pi}E_1(\mu^2 p_\pi)\left(\frac{2.3+\mu^2}{2.3-\mu^2}+\mu^2 p_\pi\right)\quad -\right.$$

$$e^{2.3p_\pi} E_1(2.3p_\pi) \left(\frac{2.3+\mu^2}{2.3-\mu^2} - 2.3p_\pi \right) - 2 \Bigg] \ ,$$

where $p_\pi = 2\alpha_\pi(t)\ln(s/M^2)/t$. $E_1(x) = -E_i(-x)$ is the exponential integral [29].

From the f-trajectory the contribution is

$$\frac{kA}{hM^2} \left[\left(\frac{s}{M^2} \right)^{-a(0)} \left(\frac{\Gamma(\gamma+1)}{(p_1^2+(\pi\delta/2)^2)^{(\gamma+1)/2}} V(s/M^2) + \right. \right.$$
$$\left. \left. \frac{C}{\sqrt{p_1^2+(\pi\delta/2)^2}} W(s/M^2) \right) + \left(\frac{s}{M^2} \right)^{-2a(0)} \frac{k}{hp_2} \right] \ ,$$

where

$$\begin{aligned} V(s/M^2) \quad &= \quad \cos\left[\frac{\pi a(0)}{2} + (\gamma+1)\tan^{-1}\left(\frac{\pi\delta}{2p_1} \right) \right] \left(\ln\frac{s}{M^2} \right) - \\ &\qquad \frac{\pi}{2}\sin\left[\frac{\pi a(0)}{2} + (\gamma+1)\tan^{-1}\left(\frac{\pi\delta}{2p_1} \right) \right], \end{aligned}$$

$$W(s/M^2) = \cos\left[\frac{\pi a(0)}{2} + \tan^{-1}\left(\frac{\pi\delta}{2p_1} \right) \right],$$

and

$$p_1 = 2b + (2\alpha'+\delta)\ln\frac{s}{M^2} \ , \qquad p_2 = 2\left(b + (\alpha'+\delta)\ln\frac{s}{M^2} \right).$$

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